Three-Phase Circuit Analysis and the Mysterious k_0 Factor

S. E. Zocholl
Schweitzer Engineering Laboratories, Inc.

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S. E. Zocholl
Schweitzer Engineering Laboratories, Inc.
Pullman, Washington USA

Abstract

The effects of load angle and fault impedance between lines and to ground are of special concern in the evaluation and application of ground distance elements. Where the symmetrical component method generally simplifies the analysis of unsymmetrical fault conditions, in these cases, a three-phase analysis may prove more convenient and versatile. This paper is a tutorial on the three-phase circuit analysis of the transmission line circuit and is of a scale that can easily be implemented on a PC using a math program such as Mathcad™ [1].

Key Words: Three-Phase Analysis, Relay Performance, Relay Settings, Fault Impedance.

Introduction

It's a stretch for most of us and some might call it a heavy dose, but this paper encourages you to turn on your computer and get fundamental about relay analysis. The subject of the paper is the three-phase circuit analysis of the transmission line circuit shown in Figure 1. It is a live document in that each step is furnished in a Mathcad 5.0 file printed as an integral part of the paper. The object of the analysis is to calculate voltage and current phasors and then use them to determine relay performance. The phasors are calculated after you enter the source impedance ratios, the voltage phase angle across the line, the fault location, and the fault impedance for the configuration shown in Figure 2. Negative-sequence directional, ground-quadrilateral, and mho-distance equations are written into the file, so you can use it to test how these characteristics are influenced by prefault load or source and fault impedance.

The self- and mutual-impedances required by the three-phase analysis are of particular interest since they are derived from the available positive- and zero-sequence line and source impedances. The derivation shows the fundamental relations between the self- and mutual-impedances (Zs and Zm) and the positive-sequence and zero-sequence impedances (z1 and z0). Consequently, the three-phase analysis is an affirmation, rather than a rejection, of symmetrical components. In the process, the ubiquitous ground compensation factor, k_0, appears unmasked as the mutual impedance, Zm.

The paper is a road map of the mathematics and fundamentals used in the analysis and includes examples of what performance to expect from negative-sequence directional, ground-mho and ground-resistance, and ground-reactance elements.

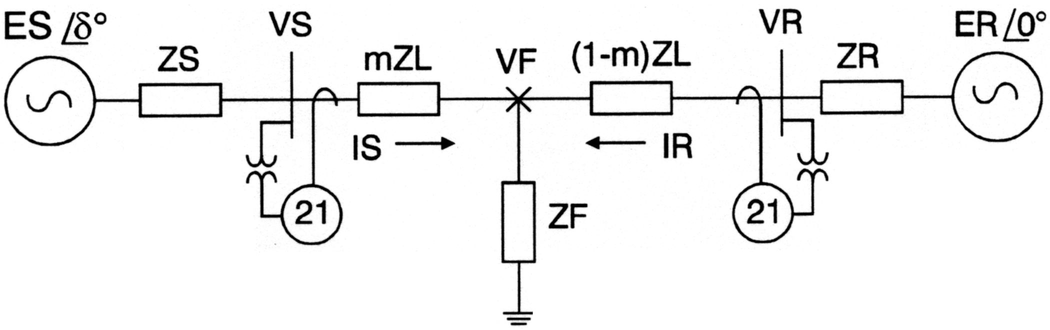

Figure 1: System One-Line Diagram

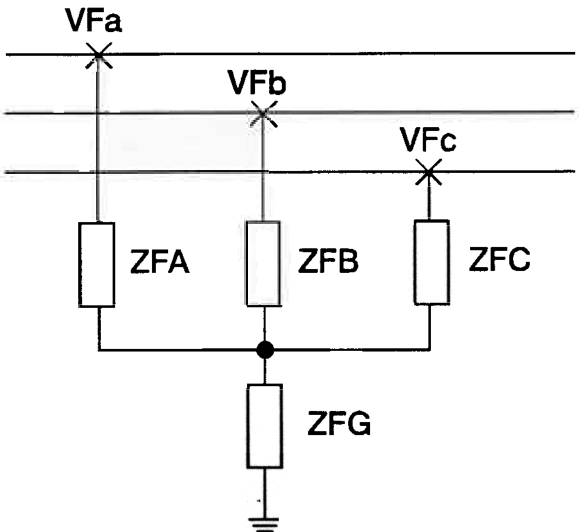

Figure 2: Fault Impedance

BUILDING THE SYSTEM MATRIX

If we were to consider the system shown in Figure 1 to be a single-phase circuit, we could write the following circuit equations:

$$\begin{aligned} ES &= ZSS \cdot IS + 0 + VF \\ ER &= +0 + ZRR \cdot IR + VF \\ 0 &= -ZF \cdot IS - ZF \cdot IR + VF \end{aligned} \tag{1}$$

where:

ES	is the source voltage
ER	is the remote source voltage
ZL	is the impedance of the transmission line
ZS	is the source impedance
ZR	is the remote source impedance
m	is the fault location in per unit of the line length
ZF	is the impedance in the fault
ZSS	= ZS + m·ZL
ZRR	= ZR + (1-m)·ZL

These equations, written with a simple circuit in mind, are valid for the three-phase system. The only difference is that each element is either a 1 by 3 column vector containing the three-phase currents and/or voltages or is a 3 by 3 impedance matrix containing self- and mutual-impedances. In matrix form they become:

$$\begin{bmatrix} \mathrm{ES} \\ \mathrm{ER} \\ \mathrm{null} \end{bmatrix} = \begin{bmatrix} \mathrm{ZSS} & \mathrm{zero} & \mathrm{one} \\ \mathrm{zero} & \mathrm{ZRR} & \mathrm{one} \\ -\mathrm{ZF} & -\mathrm{ZF} & \mathrm{one} \end{bmatrix} \begin{bmatrix} \mathrm{IS} \\ \mathrm{IR} \\ \mathrm{VF} \end{bmatrix} \tag{2}$$

where:

$$\mathrm{null} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \mathrm{zero} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \mathrm{one} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$\mathrm{ES} = \begin{bmatrix} \mathrm{ES}_a \\ \mathrm{ES}_b \\ \mathrm{ES}_c \end{bmatrix} \quad \mathrm{ER} = \begin{bmatrix} \mathrm{ER}_a \\ \mathrm{ER}_b \\ \mathrm{ER}_c \end{bmatrix} \quad \mathrm{VF} = \begin{bmatrix} \mathrm{VF}_a \\ \mathrm{VF}_b \\ \mathrm{VF}_c \end{bmatrix} \quad \mathrm{IS} = \begin{bmatrix} \mathrm{IS}_a \\ \mathrm{IS}_b \\ \mathrm{IS}_c \end{bmatrix} \quad \mathrm{IR} = \begin{bmatrix} \mathrm{IR}_a \\ \mathrm{IR}_b \\ \mathrm{IR}_c \end{bmatrix} \tag{4}$$

Although the impedance matrices contain self- and mutual-impedances, they can be expressed in terms of the positive- and zero-sequence impedances of the line or source. Consequently, the matrices ZS, ZR, and ZL can be written as the matrix function:

$$\mathrm{Z}(\mathrm{z0},\mathrm{z1}) = \begin{bmatrix} \mathrm{Zs}(\mathrm{z0},\mathrm{z1}) & \mathrm{Zm}(\mathrm{z0},\mathrm{z1}) & \mathrm{Zm}(\mathrm{z0},\mathrm{z1}) \\ \mathrm{Zm}(\mathrm{z0},\mathrm{z1}) & \mathrm{Zs}(\mathrm{z0},\mathrm{z1}) & \mathrm{Zm}(\mathrm{z0},\mathrm{z1}) \\ \mathrm{Zm}(\mathrm{z0},\mathrm{z1}) & \mathrm{Zm}(\mathrm{z0},\mathrm{z1}) & \mathrm{Zs}(\mathrm{z0},\mathrm{z1}) \end{bmatrix} \tag{5}$$

where:

z0	is the zero-sequence impedance of the line or source
z1	is the positive-sequence impedance of the line or source
Zs(z0,z1)	is the self-impedance as a function of z0 and z1
Zm(z0,z1)	is the mutual impedance as a function of z0 and z1

This function is of special interest because it establishes the link between the three-phase circuit analysis and symmetrical components. The correlation can be derived using Figure 3, which shows the self- and mutual-impedances, and the currents of a fully transposed transmission line.

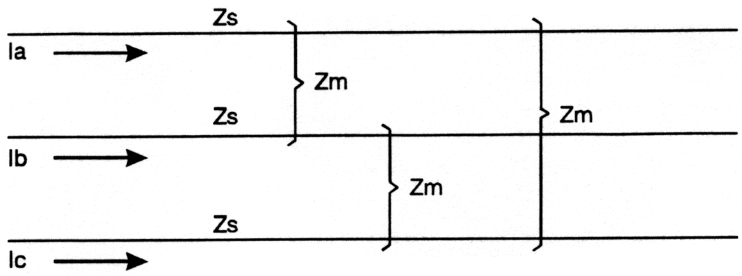

Figure 3: Line Current and Impedance

For example, in Figure 3, the voltage drop across the A-phase conductor is:

$$Va = Zs \cdot Ia + Zm \cdot (Ib + Ic) \tag{6}$$

and for the special condition of the balanced current due to a three-phase fault where $(Ib + Ic) = - Ia$:

$$Va = (Zs - Zm) \cdot Ia \tag{7}$$

Since for this condition, Va and Ia are positive-sequence quantities, it is evident that the positive-sequence impedance is:

$$z1 = Zs - Zm \tag{8}$$

Again, for another special condition where the currents are equal and in phase $(Ib + Ic) = 2\ Ia$:

$$Va = (Zs + 2 \cdot Zm) \cdot Ia \tag{9}$$

In this case, Va and Ia are zero-sequence quantities, and it is evident that the zero-sequence impedance is:

$$z0 = Zs + 2 \cdot Zm \tag{10}$$

Equations (8) and (10) can then be used to derive self- and mutual-impedance as functions of z0 and z1 for use in the impedance matrix of Equation (5). By multiplying (8) by 2, adding the result to (10), and collecting terms, we obtain the self-impedance:

$$zs(z0, z1) = \frac{z0 + 2 \cdot z1}{3} \tag{11}$$

Then, by subtracting (8) from (10) and collecting terms, we obtain the mutual impedance:

$$zm(z0, z1) = \frac{z0 - z1}{3} \tag{12}$$

Expressed as a function of z0 and z1, the mutual-impedance Zm bears a striking resemblance to k_0, the ubiquitous ground compensation factor of the residual current Ir, which enables ground distance relay elements to measure positive-sequence reach. We are now in a position to give a more direct derivation of k_0 using the general case of Equation (6) where (Ia + Ib + Ic) = Ir. To do this, we introduce Ir by adding and subtracting (Zm·Ia):

$$\begin{aligned} Va &= Zs \cdot Ia - Zm \cdot Ia + Zm \cdot (Ib + Ic) + Zm \cdot Ia \\ Va &= (Zs - Zm) \cdot Ia + Zm \cdot (Ia + Ib + Ic) \\ Va &= (Zs - Zm) \cdot Ia + Zm \cdot Ir \end{aligned} \tag{13}$$

We can now express (13) in terms of z0 and z1 using (8) and (12):

$$Va = z1 \cdot Ia + \left(\frac{z0 - z1}{3} \right) \cdot Ir \tag{14}$$

The usual form of k_0 can be seen by factoring z1 in Equation (14):

$$Va = z1 \cdot (Ia + k_0 \cdot Ir) \qquad \text{where} \qquad k_0 = \frac{z0 - z1}{3 \cdot z1} \tag{15}$$

The final essential element of the system matrix (1) is the fault impedance ZF and is defined by the impedances shown in Figure 2. This 3 by 3 matrix accounts for the voltage drop due to each phase current flowing individually in the phase-fault impedances (ZFA, ZFB, or ZFC) and mutually in the ground-fault impedance ZFG. The resulting terms are as follows:

$$ZF = \begin{bmatrix} ZFAG & ZFG & ZFG \\ ZFG & ZFBG & ZFG \\ ZFG & ZFG & ZFCG \end{bmatrix} \tag{16}$$

where:

$$ZFAG = ZFA + ZFG$$

$$ZFBG = ZFB + ZFG$$

$$ZFCG = ZFC + ZFG$$

ASSEMBLING THE SYSTEM MATRIX

When the ZSS, the ZRR, the ZF, and the unit matrices are all assembled, they form one large matrix called ZABCG. Written out, ZABCG forms a 9 by 9 array of complex numbers. Mercifully, it need not appear in this form. Instead, it is assembled in Mathcad 5.0 using the augment (A, B) function, which is an array formed by placing A and B side by side, and the stack (C, D) function, which is an array formed by placing C above D. To see these functions, look under the headings "**Build the system part of the impedance matrix**" and "**Build the fault part of the impedance matrix**." The notation is so compact that all the equations for the currents and voltages appear as follows:

The voltage vector:	E	= stack(stack(ES,ER),$(0,0,0)^T$)
Mask to get S-end current:	TS	= augment(augment(one,zero),zero)
Mask to get R-end current:	TR	= augment(augment(zero,one),zero)
Invert the impedance matrix:	YABCG	= $ZABCG^{-1}$
Solve for the fault currents:	IABCG	= YABCG·E
S-end fault current:	IS	= TS·IABCG
R-end fault current:	IR	= TR·IABCG
S-end fault voltages:	VS	= ES - ZS·IS

SPECIFYING THE FAULT CONDITION

You can specify a fault anywhere on the model line and observe the response of a relay element. As an example, the fault shown in the file is an A-phase ground fault midway on the line with a fault resistance of 0.85 ohms with no prefault current. The condition is specified by the following input:

Phase angle across line (degrees):	δ = 0.0001 (virtual zero)	
Fault location (per unit of Z1L ohms):	m = 0.5	INF = 10^{10}
Fault impedances (ohms):	ZFA = 0 + j·0	ZFB = INF + j·0
	ZFC = INF + j·0	ZGF = 0.85 + j·0

THE NEGATIVE-SEQUENCE DIRECTIONAL ELEMENT

The negative-sequence directional equations Z2F and Z2R are provided for those interested in how to set directional elements or how fault direction is declared. When using these equations, use the following convention:

Use (Z2F) if you consider the relay current to be (IS).
Use (Z2R) if you consider the relay current to be (IR).

The negative-sequence directional element measures negative-sequence impedance using the voltage and current at the relay location. The measurement is then compared to a forward- and to a reverse-impedance threshold, which are settings. The fault is declared to be forward, i.e., in front of the relay, if the measured Z2 is less than the forward-impedance threshold setting. The fault is declared to be reverse, i.e., in back of the relay, if the measured Z2 is greater than the reverse-impedance setting. The setting can be determined with the aid of Figure 4, which shows the sequence networks of the system for a line-to-ground fault at the S-bus.

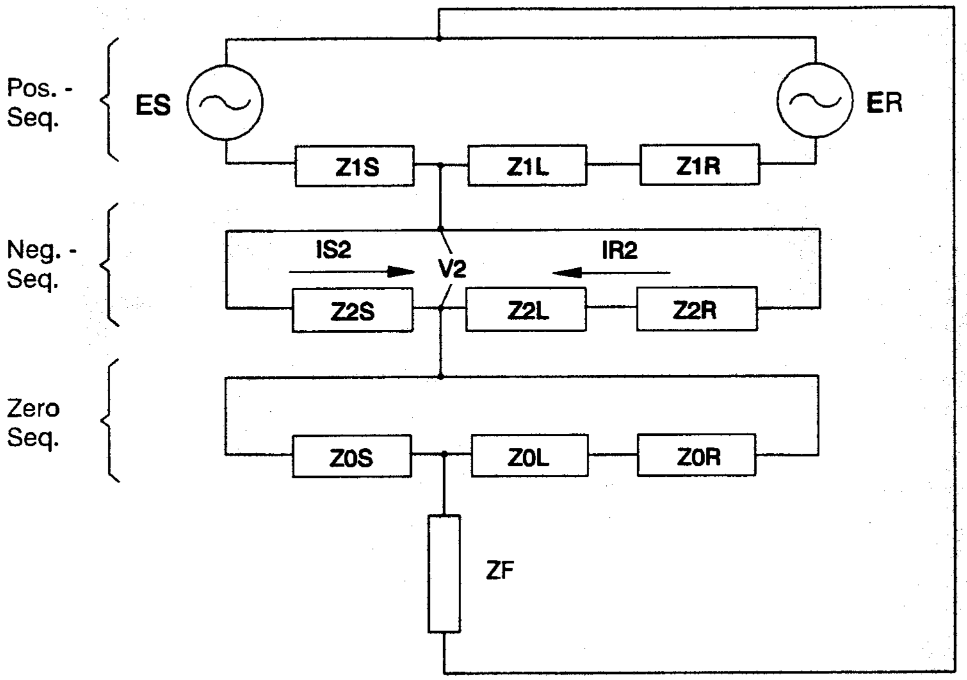

Figure 4: Sequence Network for a Line-Ground Fault

Since there are no sources in the negative-sequence network, the voltage VS2 at the S-bus is the voltage drop across the S-bus source impedance Z2S caused by the current IS2. VS2 is also the voltage drop across the impedance (Z2L + Z2R) caused by the current IR2. If the fault is in front of the relay, its input voltage is (-VS2), and its input current is (IS2). Consequently, the negative-sequence element measures the impedance (-Z2S). However, if the fault is moved in back of the relay, the current will change abruptly to (-I2R), and the directional element measures the impedance (Z2L + Z2R). The impedance span from (-Z2S) to (Z2L + Z2R) in the example is 18. Add one third of the span to (-Z2S) to get the forward-impedance threshold setting of -6 ohms. Subtract one third of the span from (Z2L + Z2R) and get the reverse-impedance threshold setting of 0.0 ohms. You will find that Z2F calculates the same 12 ohms for all faults on the line. Z2R calculates 6 ohms or greater for faults anywhere behind the relay.

THE QUADRILATERAL GROUND-DISTANCE ELEMENTS

Both ground-quadrilateral and ground-mho distance characteristic equations are provided for those interested in comparing their behavior under selected circuit conditions. The quadrilateral characteristic is bounded at the top by a reactance element and at the sides by a resistance element. The negative-sequence directional element already mentioned provides the lower boundary. All three elements must assert to produce an output.

The ground-resistance and ground-reactance characteristic equations are derived by writing the voltage drop due to an A-phase ground fault with fault resistance:

$$Va = m \cdot Z1L \cdot (Ia + k_0 \cdot Ir) + R_F \cdot I_F \tag{17}$$

This is Equation (15) except that it includes the voltage drop across the fault resistance. R_F is the fault resistance, I_F is the total fault current, and m is the fault location in per unit of the line length.

The reactance characteristic is obtained using the following process. First, make the fault resistance term real by multiplying both sides of the equation by the complex conjugate of $Ir \cdot e^{j \cdot T1}$ indicated by an overline, where T1 is the phase angle difference between the total fault current I_F and the residual current. Then take the imaginary part and solve for m:

$$m \cdot |Z1L| = \frac{\mathrm{Im}\left(Va \cdot \left(\overline{Ir \cdot e^{j \cdot T1}}\right)\right)}{\mathrm{Im}\left(\dfrac{Z1L}{|Z1L|} \cdot (Ia + k_0 \cdot Ir) \cdot \left(\overline{Ir \cdot e^{j \cdot T1}}\right)\right)} \tag{18}$$

The resistance characteristic is obtained using a similar process. First, make the line impedance term real by multiplying both sides of the equation by the complex conjugate of $Z1L \cdot (Ia + k_0 \cdot Ir)$. Then take the imaginary part and solve for the fault resistance R_F:

$$R_F = \frac{\mathrm{Im}\left(Va \cdot \left(\overline{Z1L \cdot (Ia + k_0 \cdot Ir)}\right)\right)}{\mathrm{Im}\left(I_F \cdot \left(\overline{Z1L \cdot (Ia + k_0 \cdot Ir)}\right)\right)} \tag{19}$$

This characteristic would be ideal, but I_F includes the current from both ends of the line. Instead, we must approximate the fault current by using the S-bus side negative- and zero-sequence currents in order to eliminate load current effect. The approximation is $I_F = 3/2 \cdot (IS_2 + IS_0)$.

$$R_F = \frac{\mathrm{Im}\left(Va \cdot \left(\overline{Z1L \cdot (Ia + k_0 \cdot Ir)}\right)\right)}{\mathrm{Im}\left(\dfrac{3}{2} \cdot (IS_2 + IS_0) \cdot \left(\overline{Z1L \cdot (Ia + k_0 \cdot Ir)}\right)\right)} \tag{20}$$

The equation for the reactance element as written is scaled in line ohms. The element operates when it measures less ohms than its reach setting. The resistance element operates when it observes less ohms than its reach setting. Test the reactance element to prove that its measurement is not affected by the presence of fault resistance, infeed, or prefault load current in either direction. Test the resistance element for a radial line condition to prove that its measurement is independent of prefault load current in either direction. Do this by multiplying the source impedance Z1R in the Mathcad file by the large number, INF, representing infinity. Restore the remote source Z1R to observe that the fault resistance appears amplified because the element uses only the S-side current. (Use $\delta = 25$ or $\delta = -25$ degrees to test with prefault load current.)

THE MHO GROUND-DISTANCE CHARACTERISTIC

The mho characteristic is derived using a phase comparator equation where:

$$P = \mathrm{Re}((m \cdot Z1L \cdot I - V) \cdot \overline{Vp}) \tag{21}$$

The torque product, P, is zero for any combination of I, V, and Vp that lies on a circle of reach, m. Solving for m to find the reach of the circle gives the characteristic:

$$m \cdot |Z1L| = \frac{\mathrm{Re}(VA \cdot \overline{Va1(mem)})}{\mathrm{Re}\left(\frac{Z1L}{|Z1L|} \cdot (Ia + k_0 \cdot Ir) \cdot \overline{Va1(mem)}\right)} \tag{22}$$

The equation for the mho element as written uses the positive-sequence memory voltage, Va1(mem) for Vp, and is scaled in line ohms. The element operates when it measures less ohms than its reach setting. With the line parameters given in the file, test this element with a bolted fault (ZFG = 0) at m = 0.8 and observe that the measurement is unaffected by prefault load current in either direction. To do this, set $\delta = 25$ and then to $\delta = -25$ degrees. Then add fault resistance to observe the characteristic expand, which indicates underreach.

PLOTTING THE MHO GROUND-DISTANCE CHARACTERISTIC

The mho ground-distance characteristic is plotted in Figure 3 of the Mathcad file. The memory polarized mho characteristic is perceived as being the dotted circle in Figure 3, which extends from the source impedance behind the relay to the set reach (0.8·Z1L) along the line impedance in front of the relay. Such would be the case for a three-phase fault. Note, however, that for a line-to-ground fault, the characteristic expands back to the apparent source impedance:

$$Z1S(\mathrm{apparent}) = \frac{Va1(mem) - Va}{Ia + k_0 \cdot Ir} \tag{23}$$

Consider a reach setting of 0.8, and place the fault at m = 0.5 with $\delta = 0.0001$. You can then verify that fault resistance of 0.85 ohms is on the expanded mho-characteristic circle with 0.8 reach. The Mathcad file follows:

NINEGEN.MCD

NINEGEN.MCD is a two bus power system model for general fault calculation written by Dr. E. O. Schweitzer III with generalized fault matrix contributed by Dr. John Law. The program calculates the voltage and current phasors at the local and remote bus and evaluates the equations for a negative sequence directional, and ground quadrilateral and mho distance elements. The model is shown in Figure 1 with fault impedance as in Figure 2.

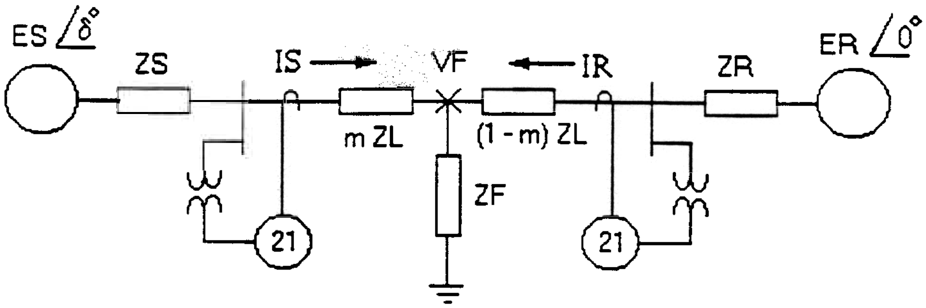

Figure 1 System One Line Diagram

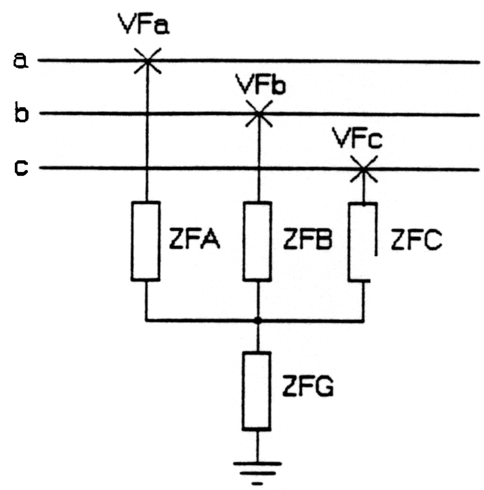

Figure 2 Fault Impedance

Program Input:

Source S Voltage:	$es := 70 \cdot e^{j \cdot \delta \cdot deg}$	Positive Seq. Line Impedance:	$Z1L := 4 \cdot e^{j \cdot 75 \cdot deg}$
Source R Voltage:	$er := 70$	Zero Seq Line Impedance:	$Z0L := 3 \cdot Z1L$
Source S SIR:	$Ssir \equiv 3 \cdot e^{j \cdot -5 \cdot deg}$	Source S Pos. Seq. Imped:	$Z1S := Ssir \cdot Z1L$
Source R SIR:	$Rsir \equiv 0.5$	Source S Zero. Seq. Imped:	$Z0S := 5 \cdot e^{j \cdot -5 \cdot deg} \cdot Z1S$
Voltage Phase Angle:	$\delta \equiv 0.001$	Source R Pos. Seq. Imped:	$Z1R := Rsir \cdot Z1L$
Fault Location:	$m := 0.5$	Source R Zero. Seq. Imped:	$Z0R := 3 \cdot Z1R$

Fault Impedances:

$ZFA := 0 + j \cdot 0$ $\quad ZFB := INF + j \cdot 0$ $\quad ZFC := INF + j \cdot 0$ $\quad ZFG := 0.85 + j \cdot 0$

Constants:

$\mathrm{rad} \equiv 1$ $\mathrm{deg} \equiv \frac{\pi}{180} \cdot \mathrm{rad}$ $a \equiv -0.5 + 0.8660254j$ $\mathrm{BAL} \equiv (1 \;\; a^2 \;\; a)^T$

$$\mathrm{one} := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathrm{zero} := \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \mathrm{INF} \equiv 10^{10}$$

Voltage Sources:

$es := 70 \cdot e^{j \cdot \delta \cdot \mathrm{deg}}$ $er := 70$

$ES := es \cdot BAL$ $ER := er \cdot BAL$

Convert positive-sequence and zero-sequence impedances to Self and Mutual Impedance:

$$zs(z0,z1) := \frac{2 \cdot z1 + z0}{3} \qquad zm(z0,z1) := \frac{z0 - z1}{3}$$

$$Z(z0,z1) := \begin{pmatrix} zs(z0,z1) & zm(z0,z1) & zm(z0,z1) \\ zm(z0,z1) & zs(z0,z1) & zm(z0,z1) \\ zm(z0,z1) & zm(z0,z1) & zs(z0,z1) \end{pmatrix}$$

$ZS := Z(Z0S, Z1S)$ $ZL := Z(Z0L, Z1L)$ $ZR := Z(Z0R, Z1R)$

Source and line impedances to the fault:

$ZSS := ZS + m \cdot ZL$ $ZRR := ZR + (1 - m) \cdot ZL$

Build the system part of the impedance matrix:

$ZTOP := \mathrm{augment}(\mathrm{augment}(ZSS, \mathrm{zero}), \mathrm{one})$

$ZMID := \mathrm{augment}(\mathrm{augment}(\mathrm{zero}, ZRR), \mathrm{one})$

$ZSYS := \mathrm{stack}(ZTOP, ZMID)$

Prefault Conditions:

$ZPRE := ZS + ZL + ZR$

$IPRE := ZPRE^{-1} \cdot (ES - ER)$

$VSP := ES - ZS \cdot IPRE$

Build the Voltage Vector:

$E := \mathrm{stack}\left(\mathrm{stack}(ES, ER), (0 \;\; 0 \;\; 0)^T\right)$

$TS := (\mathrm{augment}(\mathrm{augment}(\mathrm{one}, \mathrm{zero}), \mathrm{zero}))$

$TR := (\mathrm{augment}(\mathrm{augment}(\mathrm{zero}, \mathrm{one}), \mathrm{zero}))$

Build the fault part of the impedance matrix:

$ZFAG := ZFA + ZFG$ $\qquad$ $ZFBG := ZFB + ZFG$ $\qquad$ $ZFCG := ZFC + ZFG$

$$FABCG := \begin{pmatrix} -ZFAG & -ZFG & -ZFG & -ZFAG & -ZFG & -ZFG & 1 & 0 & 0 \\ -ZFG & -ZFBG & -ZFG & -ZFG & -ZFBG & -ZFG & 0 & 1 & 0 \\ -ZFG & -ZFG & -ZFCG & -ZFG & -ZFG & -ZFCG & 0 & 0 & 1 \end{pmatrix}$$

$ZABCG := \text{stack}(ZSYS, FABCG)$ $\qquad$ $YABCG := ZABCG^{-1}$

Fault Currents: $IABCG := YABCG \cdot E$

S-End Fault Currents: $IS := TS \cdot IABCG$

R-End Fault Currents $IR := TR \cdot IABCG$

S-End Fault Voltages $VS := ES - ZS \cdot IS$

Voltage and current naming convention for this program:

Line prefault load currents from the S Bus:

Ia =	$IPRE_0$	$\lvert IPRE_0 \rvert = 6.793 \cdot 10^{-5}$	$\arg(IPRE_0) = 18.334 \cdot deg$
Ib =	$IPRE_1$	$\lvert IPRE_1 \rvert = 6.793 \cdot 10^{-5}$	$\arg(IPRE_1) = -101.666 \cdot deg$
Ic =	$IPRE_2$	$\lvert IPRE_2 \rvert = 6.793 \cdot 10^{-5}$	$\arg(IPRE_2) = 138.334 \cdot deg$

Line prefault voltage at the S Bus

Va =	VSP_0	$\lvert VSP_0 \rvert = 70$	$\arg(VSP_0) = 3.331 \cdot 10^{-4} \cdot deg$
Vb =	VSP_1	$\lvert VSP_1 \rvert = 70$	$\arg(VSP_1) = -120 \cdot deg$
Vc =	VSP_2	$\lvert VSP_2 \rvert = 70$	$\arg(VSP_2) = 120 \cdot deg$

Line fault currents from the S Bus

Ia =	IS_0	$\lvert IS_0 \rvert = 2.426$	$\arg(IS_0) = -61.167 \cdot deg$
Ib =	IS_1	$\lvert IS_1 \rvert = 0.282$	$\arg(IS_1) = 108.006 \cdot deg$
Ic =	IS_2	$\lvert IS_2 \rvert = 0.282$	$\arg(IS_2) = 108.006 \cdot deg$

Line fault currents from the R Bus

Ia =	IR_0	$\lvert IR_0 \rvert = 9.736$	$\arg(IR_0) = -66.735 \cdot deg$
Ib =	IR_1	$\lvert IR_1 \rvert = 0.282$	$\arg(IR_1) = -71.994 \cdot deg$
Ic =	IR_2	$\lvert IR_2 \rvert = 0.282$	$\arg(IR_2) = -71.994 \cdot deg$

Symmetrical Components:

$$A := \frac{1}{3} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \qquad \begin{matrix} VSsym := A \cdot VS & ISsym := A \cdot IS & IRsym := A \cdot IR \\ VSPsym := A \cdot VSP & & VSPsym_1 = 70 + 4.072 \cdot 10^{-4} i \end{matrix}$$

Relay Constants:

$$k0 := \frac{Z0L - Z1L}{3 \cdot Z1L} \qquad T1 := \arg\left[1 + \frac{Z0S + .8 \cdot Z0L}{Z0R + (1 - .8) \cdot Z0L}\right]$$

SEL-321 Negative Sequence Directional Element:

$$Z2F := \frac{\mathrm{Re}\left(VSsym_2 \cdot \overline{\left(ISsym_2 \cdot e^{j \cdot \arg(Z1L)}\right)}\right)}{\left(\left|ISsym_2\right|\right)^2} \qquad Z2R := \frac{\mathrm{Re}\left(VSsym_2 \cdot \overline{\left(IRsym_2 \cdot e^{j \cdot \arg(Z1L)}\right)}\right)}{\left(\left|IRsym_2\right|\right)^2}$$

$Z2F = -11.954$ $\quad |Z1S| = 12$ $\qquad Z2R = 3.43$ $\quad |Z1L + Z1R| = 6$

SEL-321 Quadrilateral Ground Distance Resistance Element:

$Ir := \sum IS$

$$Irr := \sum IR \qquad RAG := \frac{\mathrm{Im}\left(VS_0 \cdot \overline{\left[\left(IS_0 + k0 \cdot Ir\right) \cdot e^{j \cdot \arg(Z1L)}\right]}\right)}{\mathrm{Im}\left[\frac{3}{2} \cdot \left(ISsym_2 + ISsym_0\right) \cdot \overline{\left[\left(IS_0 + k0 \cdot Ir\right) \cdot e^{j \cdot \arg(Z1L)}\right]}\right]}$$

$RAG = 4.603$

SEL-321 Mho and Quadrilateral Ground Reactance Element:

$$XAG := \frac{\mathrm{Im}\left(VS_0 \cdot \overline{\left(Ir \cdot e^{j \cdot T1}\right)}\right)}{\mathrm{Im}\left[\left[Z1L \cdot \left(IS_0 + k0 \cdot Ir\right)\right] \cdot \overline{\left(Ir \cdot e^{j \cdot T1}\right)}\right]} \qquad MAG := \frac{\mathrm{Re}\left(VS_0 \cdot \overline{VSPsym_1}\right)}{\mathrm{Re}\left[Z1L \cdot \left(IS_0 + k0 \cdot Ir\right) \cdot \overline{VSPsym_1}\right]}$$

$XAG = 0.5$ $\qquad MAG = 0.8$

Equation of the Mho Ground Characteristic for a Three-Phase Fault (dotted):

$i := 0..36 \qquad \phi_i := i\cdot(10)$

$$Z_i := \frac{.8\cdot Z1L - Z1S}{2\cdot|Z1L|} + \left(\left|\frac{.8\cdot Z1L + Z1S}{2\cdot|Z1L|}\right|\right)\cdot e^{j\cdot\phi_i\cdot deg}$$

Phasors to Plot:

$k := 0..1$

$$z1L_k := \begin{array}{|c|}\hline 0 \\ \hline \dfrac{Z1L}{|Z1L|} \\ \hline \end{array} \qquad Zf_k := \begin{array}{|c|}\hline 0 \\ \hline MAG\cdot\dfrac{Z1L}{|Z1L|} \\ \hline \end{array} \qquad Zaf_k := \begin{array}{|c|}\hline 0 \\ \hline \dfrac{VS_0}{(IS_0 + k0\cdot Ir)\cdot|Z1L|} \\ \hline \end{array}$$

Equation of the Mho Ground Characteristic for a Phase to Ground Fault (solid):

$$Zm_i := \frac{\left(MAG\cdot Z1L - \dfrac{VSPsym_1 - VS_0}{IS_0 + k0\cdot Ir}\right)}{2\cdot|Z1L|} + \left|\frac{MAG\cdot Z1L + \dfrac{VSPsym_1 - VS_0}{IS_0 + k0\cdot Ir}}{2\cdot|Z1L|}\right|\cdot e^{j\cdot\phi_i\cdot deg}$$

Points to Plot:

$$Zc := \frac{\left(\dfrac{.8\cdot Z1L}{|Z1L|} - \dfrac{VSPsym_1 - VS_0}{IS_0 + k0\cdot Ir}\cdot\dfrac{1}{|Z1L|}\right)}{2} \qquad Zp := -\left(\frac{VSPsym_1 - VS_0}{IS_0 + k0\cdot Ir}\cdot\frac{1}{|Z1L|}\right)$$

$$r := \frac{.8\cdot Z1L - Z1S}{2\cdot|Z1L|} \qquad Xag := XAG\cdot e^{j\cdot arg(Z1L)}$$

Graphics:

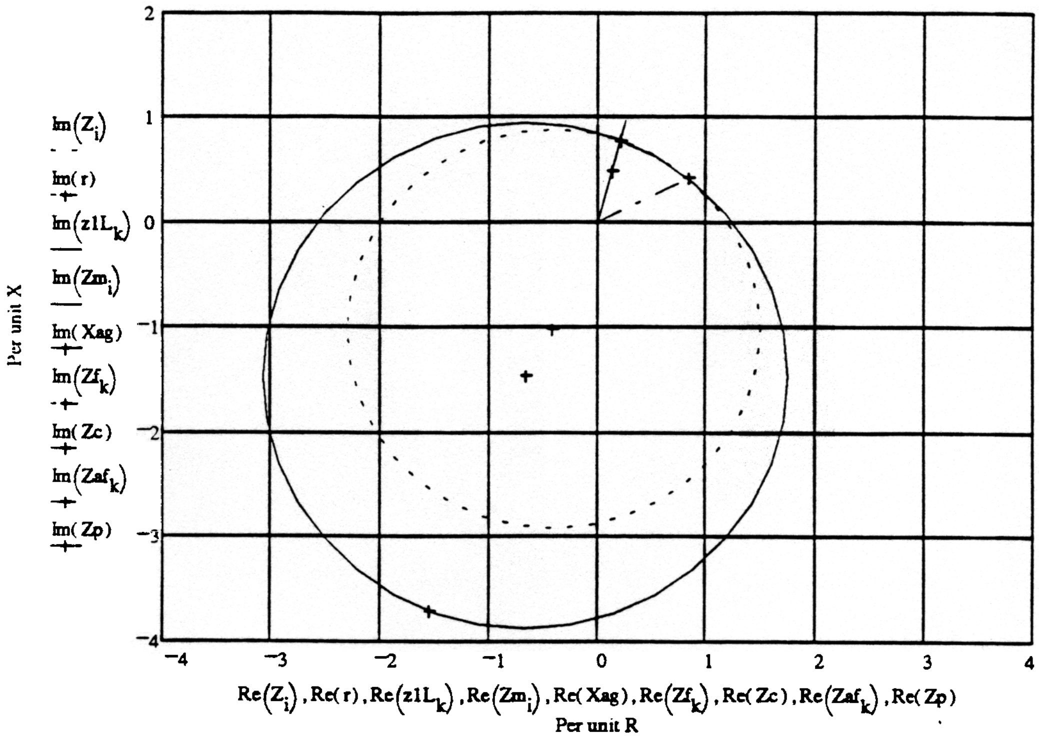

Figure 3. Plot of Mho Characteristic

Program Notes:

1. Dr. E. O Schweitzer III is the founder and President of Schweitzer Engineering Laboratories of Pullman WA. Dr. John Law is a Professor of Electrical Engineering at the University of Idaho at Moscow, ID.

2. The elements described mathematically above are covered by the following U. S Patents: 4,996,624 ; 5,041,737 ; 5,208,545 and U. S. patents pending.

CONCLUSIONS:

1. Using an automated Math Program, such as Mathcad 5.0, greatly simplifies the three-phase analysis of a power system model.

2. Where the symmetrical component method generally simplifies unsymmetrical fault analysis, a three-phase analysis may prove convenient and versatile. However, the self- and mutual-impedance required by the three-phase analysis are expressed as functions of the positive- and zero-sequence impedance. The derivation unmasks the mutual impedance as the k_0 factor Zm = (z0 - z1)/3.

3. The performance of modern distance relays is best analyzed using the calculated voltage and current phasors in the characteristic equations.

4. The object of analysis is to calculate voltage and current phasors under specified circuit conditions and to use them to determine relay performance. The analysis provides an effective means for understanding the properties of ground-directional, quadrilateral, and mho-distance elements.

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Biographical Sketch

Stanley (Stan) Zocholl has a BS and MS in Electrical Engineering from Drexel University and is an IEEE Life Fellow and a member of the Power Engineering Society. He is also a member of the Power System Relay Committee and is a past chairman of the Relay Input Sources Subcommittee. He joined Schweitzer Engineering Laboratories in 1991 in the position of Distinguished Engineer. He was with ABB Power T&D Company Allentown (formerly ITE, Gould, BBC) since 1947, where he held various engineering positions including Director of Protection Technology.

His biography appears in Who's Who in America. He holds over a dozen patents associated with power system protection using solid state and microprocessor technology and is the author of numerous IEEE and protective relay conference papers. He received the best paper award of the 1988 Petroleum and Chemical Industry Conference. In 1991, he was recognized by the Power System Relay Committee for distinguished service to the committee.