# New Multi-Ended Fault Location Design for Two- or Three-Terminal Lines

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Abstract. This paper presents a new fault location system for multi-terminal transmission lines. The algorithm used by this system is suitable for inclusion in a numerical protection relay that communicates with remote relay(s) over a protective relaying channel. Fault location estimation does not require pre-fault load flow information. In addition, the data volume communicated between relays is sufficiently small to be easily transmitted using a digital protection channel. The new algorithm does not require data alignment or phase selection information, and does not perform iterations to achieve accuracy. This new method of fault locating is not affected by pre-fault load flow, zero-sequence mutual coupling, fault resistance, power system nonhomogeneity, and current infeeds from other line terminals or tapped loads.

*Keywords*. Fault location, fault analysis, two-terminal fault location, multi-terminal fault location, numerical relaying.

#### I. INTRODUCTION

Accurate fault location information is valuable to operations personnel and power system protection engineers. Transmission line fault location has been the subject of interest to utilities and researchers since the early 1950s [1]. The reasons are obvious; accurate fault location information helps utility operations personnel expedite service restoration, thus reducing outage time, customer complaints, loss of revenue, and operating costs.

Impedance-based fault location techniques make use of fundamental frequency voltages and currents, and can be classified in two major categories. The first category is referred to as the single-ended method and makes use of fundamental frequency phasor data measured at one terminal of a transmission line. The second category is referred to as the multiterminal method and it utilizes data from more than one end of a transmission line.

Single-ended, impedance-based fault location has become a standard feature in most of today's microprocessor-based protective relays [2-5]. This fault location methodology is attractive because it is simple, fast, and does not require communications. Applications with strong zero-sequence mutual coupling, higher fault resistance, tapped loads, and nonhomogeneous power systems challenge the accuracy of single-ended fault location methods [2].

To date, there are two major single-ended fault location algorithms:

- Simple Reactance
- Takagi-based [3]

The simple reactance method works reasonably well for homogeneous systems when the fault does not involve significant fault resistance and load current. Large errors are introduced to the fault location estimate by remote-end current feed, load impedance, power transmission angle, and differing angles of line and power system source impedances.

Takagi et al [3] recognized the need to improve the simple reactance method by introducing a new single-ended method that calculates the reactance of a faulty line and provides some correction to errors caused by various factors, such as load flow and fault resistance. Schweitzer [2] recognized the limitations of both the reactance and Takagi methods, and introduced the modified Takagi algorithm. This new method improves the performance of the Takagi algorithm when some system data are available.

Takagi et al [3], and Erikson et al [4] proposed singleended fault location techniques that utilized both pre-fault and post-fault currents and post-fault voltages at one line terminal. Their technique required current distribution factors and impedances of equivalent sources behind the relay terminals. Source impedance variations due to line switching or generation variations influence the current distribution factors and introduce errors in the fault location estimate.

Two-ended impedance-based fault locating methods can improve upon the accuracy of single-ended methods. Schweitzer [2] introduced a two-ended method that did not require or assume external system parameter values. The method in [2] did not require time synchronization of the data; however, it required knowledge of pre-fault load flow information for phase alignment. Traditionally, existing two-ended methods [2, 7-9] require the phase alignment of data sets captured at both ends of a monitored line using pre-fault load flow information, iterative methods, and communication of a significant amount of data between relay terminals. Other researchers [7-10] have proposed different methodologies using fundamental frequency phasor data from two terminals of a line, and in some cases from three- or multi-terminal lines. These methods have one or more limitations, such as a requirement for data alignment, knowledge of pre-fault load flow information, need to perform iterations, and communication of a large amount of data between terminals. In addition, a number of multi-terminal methods [7,9-10] are not applicable to overhead lines with zero-sequence mutual coupling.

In this paper, we describe a new multi-ended, fault location algorithm suitable for inclusion in a protective relay where remote relay(s) communicate over a protective relaying channel. Advances of this algorithm include the following: the data required for fault location estimation do not rely on pre-fault load flow information, do not need to be synchronized, and the volume of data communicated between relays is sufficiently small to be easily transmitted using a digital protection channel. In addition, the new algorithm is not iterative, does not require phase selection information for unbalanced faults, and is very accurate. Finally, the new algorithm is not affected by pre-fault load flow and its direction, zero-sequence mutual coupling, fault resistance, power system nonhomogeneity, or current infeeds from other line terminals or tapped loads.

We next describe the two-terminal algorithm, followed by a discussion of the three-terminal fault location methodology. The new algorithm could also be extended to more than three-terminal applications. We then compare its performance with the single-ended methods for a variety of practical system conditions and line configurations. The evaluated conditions include steady-state and EMTP testing of two- and three-terminal line applications with and without system nonhomogeneity.

After demonstrating the accuracy improvements over the single-ended methods in difficult applications, we next discuss the application benefits of including such a fault locating method in a protective relay, both as part of an integrated substation environment, and as part of a power system protection engineer's desktop. These benefits include accurate fault location reporting and control of reclosing relay and mid-line motor-operated switches.

#### II. NEW APPROACH TO MULTI-ENDED FAULT LOCATION

#### A. Two-Terminal Line Application

The proposed new method of locating unbalanced faults uses negative-sequence quantities from all line terminals. By using negative-sequence fault quantities, we overcome the difficulties associated with pre-fault load flow, overhead line zero-sequence mutual coupling effects, and zero-sequence current infeeds from tapped loads along the transmission line. Further, this method of fault location for two-terminal lines does not require alignment of the data sets. Data alignment is not required because the algorithm employed at each line end uses the following quantities from the remote terminal (which do not require phase alignment):

- Magnitude of neg.-seq. current, |I<sub>2</sub>|.
- Calculated neg.-seq. source impedance,  $Z_2 \angle \theta_2^{\circ}$

Consider the sequence connection diagram shown in Fig. 1. A fundamental observation from Fig. 1 is that the negative-sequence fault voltage  $(V_{2F})$  is the same when viewed from all ends of the protected line.

At Relay S: 
$$V_{2F} = -I_{2S} \cdot (Z_{2S} + m \cdot Z_{2L})$$
 (1)

At Relay R: 
$$V_{2F} = -I_{2R} \cdot (Z_{2R} + (1 - m) \cdot Z_{2L})$$
 (2)

2



Fig. 1. Connection of Sequence Networks for a Single Line-to-Ground Fault at m

Eliminate  $V_{2F}$  from Equations 1 and 2 and rearrange the resulting expression as follows:

$$I_{2R} = I_{2S} \cdot \frac{(Z_{2S} + m \cdot Z_{2L})}{(Z_{2R} + (1 - m) \cdot Z_{2L})}$$
(3)

To avoid alignment of Relay S and R data sets, take the magnitude of both sides of Equation 3 as follows:

$$\left| I_{2R} \right| = \left| I_{2S} \cdot \frac{\left( Z_{2S} + m \cdot Z_{2L} \right)}{\left( Z_{2R} + (1 - m) \cdot Z_{2L} \right)} \right|$$
(4)

Equation 4 is then simplified to Equation 5 below.

$$|I_{2R}| = \frac{|(I_{2S} \cdot Z_{2S}) + m \cdot (I_{2S} \cdot Z_{2L})|}{|(Z_{2R} + Z_{2L}) - m \cdot (Z_{2L})|}$$
(5)

To further simplify Equation 5, define the following variables:

$$\begin{split} I_{2S} \bullet Z_{2S} &= a + j b \\ I_{2S} \bullet Z_{2L} &= c + j d \\ Z_{2R} + Z_{2L} &= e + j f \\ Z2L &= g + j h \end{split}$$

Substituting these variables into Equation 5 produces:

$$\left|I_{2R}\right| = \frac{\left|\left(a + jb\right) + m \cdot \left(c + jd\right)\right|}{\left|\left(e + jf\right) - m \cdot \left(g + jh\right)\right|}$$
(6)

Taking the square of both terms of Equation 6, expanding and rearranging terms produces a quadratic equation of the form:

$$\mathbf{A} \cdot \mathbf{m}^2 + \mathbf{B} \cdot \mathbf{m} + \mathbf{C} = 0 \tag{7}$$

Equation 7 is now easily solved for m using a quadratic solution. The coefficients of Equation 7 are given below.

$$A = |I_{2R}|^{2} \cdot (g^{2} + h^{2}) - (c^{2} + d^{2})$$
  

$$B = -2 \cdot |I_{2R}|^{2} \cdot (e \cdot g + f \cdot h) - 2 \cdot (a \cdot c + b \cdot d)$$

$$C = |I_{2R}|^{2} \cdot (e^{2} + f^{2}) - (a^{2} + b^{2})$$
(8)

From the above mathematical derivation, we show that each relay at each line terminal of the protected two-terminal line must transmit a minimal amount of information. The minimal information sent by Relay S for a two-terminal application is:

- 1. Magnitude of neg.-seq. current,  $|I_{2S}|$
- 2. Magnitude of neg.-seq. source impedance,  $|Z_{2S}|$
- 3. Angle of neg.-seq. source impedance,  $\angle Z_{2S}^{\circ}$

Using this information combined with the negativesequence quantities measured by each relay, we can solve for the fault location at each terminal without iterations.

## III. THREE-TERMINAL LINE APPLICATION

Many times utilities connect another line with a positivesequence source to an existing two-terminal line. This creates a three-terminal line, i.e., a line with three sources, which can contribute to the energy of a fault. Utilities are motivated to do this for many reasons. The most compelling reasons are voltage support and increased operational flexibility. Such lines are much more complex to protect using conventional distance and directional protection schemes. These same lines present great difficulty in the task of fault location. Fig. 2 shows a double-circuit three-terminal transmission line arrangement.

Let us assume that we have fault locating devices at each end of a transmission line. For a single-line-ground fault on Line 1 in Fig. 2, the relays at terminals 1, 2, and 5 operate to clear the fault and at the same time exchange a minimal amount of information over a protection channel for the purpose of accurate fault location. Each relay determines the faulted line section, reduces the three-terminal line arrangement into a two-terminal equivalent, and applies the twoterminal methodology described earlier to calculate the location of the fault.



Fig. 2 Typical Double-Circuit Three-Terminal Application

The relays convert any three-terminal line arrangement into a two-terminal line equivalent by observing the following points in the sequence connection diagram of Fig. 3. Note that the parallel three-terminal line was omitted to simplify the diagram and the explanation of the algorithm without loss of accuracy.



Fig. 3 Sequence Network Connection for Three-Terminal System

The sequence connection diagram in Fig. 3 assumes a single-line-ground fault located m per-unit distance from Bus X, which is connected to Source X. With the fault on Line 1, we observe that the negative-sequence voltage at the tap can be calculated by Relays 2 and 5 and the calculated tap voltage, V2TAP, is the same, if we assume for the moment that the relays sample synchronously. Fault location knowledge on Line 1 is not required to accurately calculate V2TAP at Relays 2 and 5. Instead, the required parameters are the negativesequence line impedances from Relay 2 and Relay 5 to the tap point, and the negative-sequence voltage and current phasors measured by the relays.

Each relay calculates V<sub>2TAP</sub> as follows:

At Relay 1: 
$$V_{2TAP@1} = V_{2@1} - Z_{2L1} \cdot I_{2@X}$$
  
At Relay 2:  $V_{2TAP@2} = V_{2@2} - Z_{2L2} \cdot I_{2@X}$ 

It Relay 2: 
$$V_{2TAP@2} = V_{2@2} - Z_{2L2} \cdot I_{2@Y}$$

At Relay 5:  $V_{2TAP@5} = V_{2@5} - Z_{2L5} \bullet I_{2@Z}$ 

For faults on Line 1,  $|V_{2TAP@2}| = |V_{2TAP@5}|$ . Each relay calculates  $V_{2TAP}$  and transmits this information to the remote terminals. Once each relay receives the calculated tap voltage from the other two terminals it proceeds with the identification of the faulted section by comparing the magnitudes of  $|V_{2TAP@1}|$ ,  $|V_{2TAP@2}|$ , and  $|V_{2TAP@5}|$ , along with the calculated single-ended fault location values. The faulted line section is the one whose V2TAP voltage magnitude does not equal the  $V_{2TAP}$  calculated by the other two relays. In the example shown in Fig. 3,  $|V_{2TAP@2}|$  and  $|V_{2TAP@5}|$  have the closest magnitude match.

Once the fault location system identifies the faulted line section, Relay 2 and Relay 5 calculate an apparent negative-sequence source impedance by the parallel combination of  $(Z_{2L2} + Z_{2Y})$  and  $(Z_{2L5} + Z_{2Z})$ . This conversion is simply  $V_{2TAP}/(I_{2@Y} + I_{2@Z})$ .  $I_{2@Y}$  and  $I_{2@Z}$  from Relays 2 and 5 cannot be added directly without phase alignment because the relays

sample the power system analog quantities asynchronously. The alignment angle between Relays 2 and 5 is simply calculated as the  $\angle$ (V<sub>2TAP@2</sub>/V<sub>2TAP@5</sub>). After determining this angle, Relay 2 phase shifts the negative-sequence current from Relay 5 and adds the result to its own negative-sequence current. Relay 5 performs a similar calculation, and both relays then calculate an apparent negative-sequence source impedance from the tap looking into the power system.

The minimal information sent by each relay to each of the other relays in the three-terminal application is:

- 1.  $|I_{2RELAY}|$
- 2.  $\angle I_{2RELAY}$
- 3.  $|V_{2TAP}|$
- 4.  $\angle V_{2TAP}$

From these transmitted quantities, each relay performs the following steps before calculating the fault location.

- Compare the magnitudes of  $V_{2TAP}$ . Those relays with approximately the same  $|V_{2TAP}|$  are not associated with the faulted line section. Call these relays Remote 1 and 2.
- From Remote 1 and 2 ∠V<sub>2TAP</sub> values, calculate the alignment angle between these relays.
- Adjust the angle of the adjacent remote relay negativesequence current by the alignment angle calculated in the previous step.
- Add the measured negative-sequence current with the phase shifted negative-sequence current of the selected remote relay. Call this summation current as I<sub>2TAP</sub>.
- Calculate  $Z_{2TAP}$  as  $V_{2TAP} / I_{2TAP}$ .

The network reduction described earlier allows the use of the algorithms developed for the two-terminal line application. Thus, for three-terminal lines the relays perform the following substitutions:

Three-terminal:  $Z_{2TAP} + Z_{2L} = e + jf$  $I_{2R} = I_{2TAP}$ 

Note that all relays have the calculated  $V_{2TAP}$  from the other relays, and each relay then knows which information to use to calculate an accurate fault location. After each relay calculates its multi-ended fault location, it calculates the total fault current and fault resistance RF. The power system operator can then interrogate any relay to determine the correct fault location.

#### IV. TESTING AND RESULTS OF THE NEW ALGORITHM

The algorithm presented above has been tested extensively using steady-state and transient methods. The next two sections of the paper describe one power system model and the results of this algorithm and comparisons to single-ended algorithms.

#### A. Steady-State Testing

Fig. 2 shows the example three-terminal system we modeled to test the algorithm. Table 1 shows the results of the new method as compared to single-ended methods. Legend:

I<sub>2P</sub> Single-Ended Negative-Sequence Current Polarized Fault Locator

X,Y,Z Terminals X, Y, and Z

From the data in Table 1, we see that the new method of fault location described in this document out-performs the single-ended method. The new method was also compared with a single-ended method using zero-sequence current polarization and the results are similar to the ones shown in Table 1. The performances of both single-ended methods are influenced by zero-sequence mutual coupling and system non-homogeneity. To see the effect of nonhomogeneity, notice that the error in both methods increases as the fault location is moved away from the relay location. Also, notice that the single-ended fault locations at terminals 2 and 5 have significant errors for faults near Bus X.

TABLE 1

FAULT LOCATION RESULTS FOR A THREE-TERMINAL LINE				
Fault on Line	New	Relay-1	Relay-3	Relay-5
1 m p.u. from	Algorithm	with I2p	with I2p	with I2p
Terminal X	at R-1	at X	at Y	at Z
0.0	0.0	0.00	14.74	13.63
0.1	0.1	0.09	12.24	11.50
0.2	0.2	0.18	10.41	9.86
0.3	0.3	0.27	8.89	8.47
0.4	0.4	0.36	7.56	7.23
0.5	0.5	0.45	6.34	6.09
0.6	0.6	0.54	5.20	5.00
0.7	0.7	0.62	4.10	3.96
0.8	0.8	0.70	3.05	2.96
0.9	0.9	0.77	2.03	1.97
1.0	1.0	0.75	1.02	1.01

# B. Electromagnetic Transient Program Testing

Fig. 4 shows one power system we modeled with the Electromagnetic Transient Program (EMTP) to test the new algorithm. The system consists of two sources and two parallel, mutually coupled 500 kV lines. The 500 kV lines from Bus RMT5 to Bus TMT5 are 100 miles long and are modeled with four 25-mile sections of frequency-dependent line models. We simulated various types of faults, varying the load flow magnitude and direction, fault resistance, and fault incidence angle.



Fig. 4 EMTP Model of a Two-Source System

The transient data generated by EMTP were converted to COMTRADE format and processed with a software program that models the fault location algorithm, associated logic, and ability to introduce varying amounts of phase shift to the data from one relay terminal to simulate the effects of asynchronous sampling.

Fig. 5 and Fig. 6 show results of the new algorithm for a line-to-ground fault at 75% from Bus RMT5, and comparison

with a single-ended fault location algorithm. The software program implementation of fault location for a personal computer differs from that of a protective relay. With this program, we calculate a fault location for each line of data sampled by both relays. In a relay implementation, the relay selects an optimal set of data and transmits it to the other end. Computer simulations such as the ones shown here are used to design the necessary logic within a microprocessor relay, and to develop the search algorithms for selecting the optimal time after fault inception to send the negative-sequence quantities described earlier.



Fig. 5 Fault Location from Relay 1 (Ideal=75)

Fig. 7 shows the calculation of fault resistance by Relays 1 and 2. The fault resistance is displayed in secondary ohms. The ct and ccvt ratios are 400/1 and 4330/1 respectively. The primary fault resistance used in this example was 30 ohms (2.77 W secondary).



Fig. 6 Fault Location From Relay 2 (Ideal = 25)



Fig. 7 Secondary Fault Resistance Calculation

### V. APPLICATIONS OF FAULT LOCATOR OUTPUTS

The multi-ended fault locator described in this paper outputs the fault location (FL), fault resistance (RF), total fault current (IFLT) as a minimum. The FL output has an immediate usefulness in informing the system operators where to direct line crews for repairing the power line and restoring service to the customers quickly. The following text describes an additional use of FL for near real-time power system control and configuration.

In Fig. 8, Sources S and R serve Loads A and B, respectively. Only the breakers at each line end are rated to interrupt fault current. The switches shown, SW1–SW4, are rated to switch load and line charging current. For the fault location shown between SW2 and SW3, Relays S and R detect the fault and trip their associated breakers.

One traditional line control scheme requires that SW1– SW4 trip when the voltage on either side of the switch is near zero for a set time. The reclosing scheme then closes a breaker to test the line. If the line tests OK, then the control scheme at each switch closes to test the next portion of the line. If a line section tests bad, the operator knows the faulted line section. Let us look at the example control actions from the perspective of Relay S given the fault shown in Fig. 8.

- Following the initial trip, all switches open according to the line and bus undervoltage detection scheme at each switch.
- After the short coordinating delay, Relay S closes the breaker near Source S. This line test should be OK.
- SW1 closes (SW2 is still open). This energizes Load A.
- SW2 closes into the permanent fault.
- Relay S trips its breaker.
- Switches 1 and 2 again open according to their line and bus undervoltage trip scheme.

- After the short coordinating delay, Relay S again closes the breaker near Source S. This line test should be OK.
- SW1 closes (SW2 is open and locked out). This again energizes Load A.

The drawback of this scheme is that it unduly penalizes those customers represented as Load A. Relays can compare the FL output against two thresholds representative of the distance between SW2 and SW3 (call these thresholds 10 and 30 to represent line distance from Bus S to SW2 and SW3 respectively). If 10 < FL < 30, the output of the comparator logic is a logical 1. If the fault location is measured between SW2 and SW3, the relay can automatically send trip commands to SW2 and SW3 via a communications channel. This scheme improves power system operation by restoring Loads A and B very quickly since this approach does not require time coordinating delays, nor does it use the process of elimination to locate the faulted line section.



a. Faulted System Between SW2 and SW3



b. Traditional Control Scheme Action for Fault Between SW2 and SW3 De-energizes Loads A and B



c. Improved Control Scheme Action for Fault Between SW2 and SW3 Does Not Interrupt Load Service to Load A and B

Fig. 8 Application of Fault Location Output

#### VI. CONCLUSIONS

Single-ended fault location methods can be accurate if the power system is fairly homogeneous, and the mutual coupling between parallel transmission lines in the zero-sequence network is weak.

Multi-ended fault location algorithms can greatly improve the fault location accuracy. Many of the existing algorithms require the transfer of large amounts of data, alignment of the data sets, and iterative solutions to calculate the distance to the fault point. This makes their application limited to processing the data offline and adds considerable amount of time in the fault location process. In addition, some of the existing twoterminal methods cannot adequately handle mutual coupling and tapped loads with zero-sequence current infeeds, and are not applicable to more than two-terminal lines.

The new algorithm presented in this paper has many advantages over existing methods in that it can calculate the location of the fault in nearly real time, it is immune to mutual coupling and tapped loads, and does not require data alignment or prefault load information. It is also applicable to three-terminal lines with great accuracy (error rate of less than one percent). In addition, the new algorithm calculates the fault resistance and the total current in the fault path. The ability of the relays to calculate the actual fault location and present the data to operations personnel make it very attractive, since operators do not have to use off-line computer programs to process and analyze event reports after the occurrence of a fault.

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# VIII. APPENDIX

Data for the example system of Fig. 2: Source X, Y, and Z Data: Positive-sequence: Source X:  $Z1 = 5.648 \ \Omega \angle 76.5^{\circ}$ Source Y: Z1 = 1.06  $\Omega \angle 90^{\circ}$ Source Z:  $Z1 = 0.275 \ \Omega \angle 90^{\circ}$ Zero-sequence: Source X Z0 = 28.52  $\Omega \angle 77.139^{\circ}$ Source Y Z0 =  $1.108 \ \Omega \angle 90^{\circ}$ Source Z Z0 =  $0.278 \ \Omega \angle 90^{\circ}$ Line data: Line 1 Length = 20 miles (Line 3 = Line 1) Line 2 Length = 7 miles (Line 4 = Line 2) Line 5 Length = 5 miles (Line 6 = Line 5) Positive-sequence: Line 1 Z1 = 15.773  $\Omega \angle 77.2^{\circ}$ Line 2 Z1 = 5.521  $\Omega \angle 77.2^{\circ}$ Line 5 Z1 = 2.366  $\Omega \angle 77.2^{\circ}$ Zero-sequence: Line 1 Z0 = 49.434  $\Omega \angle 80.1^{\circ}$ Line 2 Z0 = 17.302  $\Omega \angle 80.1^{\circ}$ Line 5 Z0 = 7.415  $\Omega \angle 80.1^{\circ}$ Line Mutual Data: Line 1–3 ZM = 30.483  $\Omega \angle 79.2^{\circ}$ Line 2–4 ZM = 10.669  $\Omega \angle 79.2^{\circ}$ Line 5–6 ZM =  $4.572 \ \Omega \angle 79.2^{\circ}$